International Journal of Theoretical Physics, Vol. 23, No. 10, 1984

Gravitation and Irreversibility

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Received September 12, 1983

The internal time operator M is an alternative to the usual dynamic time, an independent parameter of motion. Even when the dynamical entity is the three-geometry ${}^{(3)}G$ and we are concerned with its evolution in superspace (the problem of cosmological evolution), dynamical time remains an independent parameter associated with a choice of lapse and shift functions. The quantity M is, on the contrary, an ensemble-dependent parameter related to the "age" of a process: the entirety of the ensemble's evolution. With this different view of time as age, we seek a geometrical counterpart to M for the ${}^{(3)}G$ as an "ensemble." For a closed negatively curved universe, a Lyapounov function can be established which allows an M to be defined for the Robertson–Walker universe. The time component of superspace momentum π_{uv} is introduced, and we identify its conjugate energy $\partial S/\partial\pi_{uv}$ with dissipation due to the evolving universe. A geometrical counterpart of M is introduced by a conformal invariant Γ. This quantity simultaneously expresses (i) the topological feature of orientation-preserving transformations, and (ii) the Hamiltonian treatment of dissipative systems. This dual character of Γ, which links topological change to dissipative systems, suggests a geometrical basis for M. In this sense irreversibility is incorporated into the geometric structure of space-time, along with gravitation.

1. INTRODUCTION

There is a fundamental link between "internal time" and cosmology. Specifically, an internal time operator has recently been introduced to characterize the evolution of the Robertson–Walker universe (Lockhart et al., 1982). This internal time differs profoundly from proper time—dynamical time—as measured by an external clock moving with a particle. In this paper, we will examine the *geometrical* significance of internal time and its contrast with clock time.

The internal time operator acts on the phase-space distribution function and is canonically conjugate to the Liouville generator of motion. From

0020-7748/84/1000-1009$03.50/0

this operator, one can associate an age with the distribution function which corresponds to a decreasing H function. Thus, irreversibility has its microcanonical basis in the internal time. Further, the time variable, $\lambda(t)$, associated with internal time is a nonlinear function of clock time t. It has been introduced into cosmology because it provides the possibility of avoiding the problem of a singularity due to gravitational collapse predicted by Einstein's theory. Proper clock time is not well defined near the singularity, and in the words of Lockhart et al. (1982).

> It seems more appropriate to introduce a time concept which, unlike proper time, does not involve the time measured by an external clock carried by an observer or particle, but is related to the intrinsic properties of the motion of the particle itself. In other words, the new time concept we seek should refer in some suitable sense to the "*internal time*" associated with the particle's motion.

Internal time focuses attention on the age of a process, such as the evolution of an ensemble, and thus emphasizes a view of time as duration—an indivisible whole. There are deep philosophical issues associated with this view, issues evoking the names of Whitehead, Bergson, and Heraclitus. Prigogine (1980) has characterized the view as the "physics of becoming." Rooted in the phenomenological basis of thermodynamics, the concept of internal time is actually broader and stands complementary to the "physics of being": classical, relativistic, and quantum physics. It is a fresh conception of time offering opportunities to reexamine critical problems. As Wheeler (1982) says "... in the profound issues of principle that confront us today, no difficulties are more central than those associated with the concept of time."

As the present work investigates the link between internal time and cosmology, it respects the tradition of *physics as geometry*. The evolution of scientific thought leading to Einstein's geometric theory of gravity may be extended to encompass irreversibility if there is the possibility of any geometrical significance in internal time. What would be the features of, for example, a geometric counterpart to the age of the universe? This geometric feature of the manifold would express cosmological lifetime and the irreversible evolution of the universe. How would this geometric internal time differ from the conception of dynamical time in general relativity?

In the following section we review the application of the internal time operator to cosmological evolution for the specific case of the Robertson–Walker model. Next this is contrasted with a review of dynamical time in general relativity and the evolution of space-time governed by Einstein's law. We then return to discuss internal time, irreversibility, and dissipation. A key development links dissipative structures to topological change, and we conclude with a geometric (topological) basis for time as age.

2. COSMOLOGICAL INTERNAL TIME

The key ideas which allow an internal time operator M to be constructed for a cosmological model are (i) geodesic flows on a compact manifold of negative curvature are known to be K flows (Sinai, 1960), and (ii) geodesic flow on a four-manifold can be reduced to geodesic flow on a three-manifold under special conditions. *Geodesic flow* carries every line element along the geodesic which defines the line element at a point. For a Riemannian volume element dV and differential $d\theta$ determined by geodesic directions, the invariant measure $d\mu$ in a geodesic flow is $d\mu = dV d\theta$. Geodesic flow on a four-manifold may be reduced to geodesic flow on a three-manifold by utilizing the Robertson—Walker metric:

$$g_{00} = -1, \qquad g_{ij} = R^2(t)\gamma_{ij}$$

$$\gamma_{ij} = \left\{1 + \tfrac{1}{4}k\left[(x^1)^2 + (x^2)^2 + (x^3)^2\right]\right\}^{-2}\gamma_{ij} \tag{1}$$

where i, $j = (1,2,3)$. Lockhart, Misra, and Prigogine (1982) have shown that the three spatial coordinates $x^i(\sigma)$ of unconstrained particles follow geodesic motion in the three-dimensional hypersurface with metric γ_{ij}. The resulting three-surface, with negative curvature, may be compactified to give a nonstandard cosmology in the sense that the space-time is not globally isotropic but is locally isotropic.

Geodesic flows on a compact manifold of constant negative curvature are K flows (Wolf, 1977). K flows are flows which exhibit a high degree of instability. For example, the distance between adjacent geodesics increases exponentially. Thus, if $L(t) = [g_{ij}\Delta x^i(t)\Delta x^j(t)]^{1/2}$ is the general spatial distance between geodesics of particles moving with respect to the cosmological fluid then

$$L_0(t) = \left[\gamma_{ij}\Delta x^i(t)\Delta x^j(t)\right]^{1/2} = L(t)/B(t) \tag{2}$$

is the distance between geodesics projected in the fixed three-dimensional hypersurface with metric γ_{ij} [where $B(t)$ is a time dependent scale factor]. Change of variable with the affine parameter $\lambda, t \to \lambda(t)$, gives

$$\dot{L} = dL/dt = (\dot{B}/B)L + (c\dot{\lambda})L \tag{3}$$

where c is a positive constant. The second term is due to geodesic instability based upon negative curvature. The evolving distribution of particles in the universe thus contains an element of instability characterizing K flow.

K-flow instability permits the introduction of a variable, T (related to a *Lyapounov* function), which may be interpreted as "internal cosmological time." It is a nonlinear function $\lambda(t)$ of proper clock time t. T satisfies the relationship

$$[T, U_\lambda] = \lambda U_\lambda \tag{4}$$

where U_λ is the unitary group induced by the above-mentioned projection. It has the property

$$U_\lambda^* T U_\lambda = T + tI \tag{5}$$

which expresses the feature that average internal time $\langle T \rangle$ given by

$$\langle T \rangle_\rho = \int_\Omega \bar{\rho}(T\bar{\rho})\,d\mu = \langle \bar{\rho}, T\bar{\rho} \rangle \tag{6}$$

(ρ being the ensemble distribution function) advances with increasing clock time t.

Describing cosmological evolution with the internal time operator T acting on distribution functions introduces thermodynamic considerations. The average $\bar{c}$ of the positive constant c appearing in the geodesic separation $L(t)$ is equal to the K entropy of the geodesic flow. In the following section we consider cosmological evolution from the perspective of dynamical (clock) time. This is the evolution of the universe as governed by Einstein's law.

3. COSMOLOGICAL EVOLUTION ACCORDING TO GENERAL RELATIVITY

Einstein's field equation $G_{uv} = c^4/GT_{uv}$ couples the distribution of mass-energy, which is expressed by the tensor T_{uv}, to the geometry of space-time which is expressed by the curvature tensor G_{uv}. This geometric feature of the manifold, its curvature, is physically manifest in the phenomenon of gravitation. In the words of Wheeler (Misner et al., 1970), "...matter here curves space here... To produce a curvature in space here is to force a curvature in space there,... Thus, matter here influences matter there." The incorporation of gravity into the geometry of space-time validates a lengthy intellectual history of *physics as geometry*.

Einstein's equation also governs the evolution of the distribution of mass-energy and, accordingly, the evolution of the geometry of space-time.

The curvature of space-time is therefore not static. Geometry poses a problem in dynamics. In a formulation of *geometrodynamics* (Wheeler, 1962) associated with Arnowitt, Deser, and Misner (1962), and also Kuchar (1972), one specifies an initial configuration of space-time geometry and a final configuration. The intermediate configurations which are dynamically allowed by Einstein's law specify the evolving geometry. This extremum approach to general relativity is based upon the variational principle first described by Hilbert. The action integral

$$I=\int \mathscr{L} d^4x=\int(-g)^{1/2} L d^4x \tag{7}$$

involves a Lagrangian which depends upon the scalar curvature invariant R in the fashion

$$L=(c^3/16\pi G)R \tag{8}$$

The extremal principle $\delta I=0$, gives the action $S=I$ (extremum).

The result of investigations (Dirac, 1959; Wheeler, 1964; Misner, 1972) is that the dynamic entity of general relativity is not four-dimensional space-time (as one might expect), but the three-geometry ${}^{(3)}G$ with metric g_{ij} $(i, j=1,2,3)$. A *three-geometry* is the equivalence class of diffeomorphic spacelike slices for an event P: a class of three-metrics g_{ij} which are equivalent to each other under coordinate transformations. The action S is extremized between initial and final three-geometries ${}^{(3)}G$. For a "thin sandwich" of initial and final three-geometries, ${}^{(3)}G_1$ and ${}^{(3)}G_2$, the relative spacing and orientation of the two hypersurfaces is expressed by lapse and shift functions, N and N_i, respectively. These lapse and shift functions relate the projection of the three-metric g_{ij}, to the four-metric g_{uv}, such that

$$g_{ij}(dx^i+N^i dt)(dx^j+N^j dt)-(N dt)^2=g_{uv}\,dx^a dx^b \tag{9}$$

and

$$\begin{Vmatrix} g_{00} & g_{0k} \\ g_{i0} & g_{ik} \end{Vmatrix}=\begin{Vmatrix} (N_s N^s-N^2) & N_k \\ N_i & g_{ik} \end{Vmatrix} \tag{10}$$

They allow an extrinsic curvature K_{ij} of the three-geometry to be expressed as

$$K_{ij}=(1/2N)(N_{i|j}+N_{j|i}-\partial g_{ij}/\partial t) \tag{11}$$

A geometrodynamic field "momentum" π_{ij} which is conjugate to the geometrodynamic field "coordinates" g_{ij} of the ${}^{(3)}G$ can also be defined as

$$\pi_{ij} = \partial S / \partial g_{ij} \tag{12}$$

A Hamiltonian can be formed from these coordinates and momenta, $H(\pi^{ij}, g_{ij})$ and its covariant derivative is

$$H^i\left(\pi^{ij}, g_{ij}\right) = -2\pi^{ij}|j \tag{13}$$

According to Arnowitt, Deser, and Misner, the action principle Lagrangian becomes, in the above terms,

$$\mathscr{L} = -g_{ij}\partial\pi^{ij}/\partial t - NH - N_i H^i + \mathscr{L}_{\text{fields}} \tag{14}$$

The first term may be reexpressed

$$-g_{ij}\partial\pi^{ij}/\partial t = -\partial/\partial t\left(g_{ij}\pi^{ij}\right) + \pi^{ij}\partial g_{ij}/\partial t \tag{15}$$

to focus attention on variation of the three-metric g_{ij} and the full time derivative of the first term. At this point, a very important abridgement is made. The variation of action S depends only upon a variation in the three-geometry,

$$\delta S = \int \pi^{ij}\delta g_{ij}\, d^3x \tag{16}$$

since the dynamic entity is the ${}^{(3)}G$. The full time derivative is dropped from the Lagrangian because such a time variable is supplemental to the proper time separation of initial and final hypersurfaces expressed in the extrinsic curvature K_{ij} of the ${}^{(3)}G$'s. No additional time variable is required in the dynamics as usually posed, so the first term of equation (14) is neglected. Thus, "all the information about time relevant to general relativity is contained in the three-geometry" (Misner et al., 1970).

Contrasting the action variational (16) of general relativity with the corresponding variational from elementary mechanics

$$\delta S = p\,\delta x - E\,\delta t \tag{17}$$

(where the momentum p corresponds to π^{ij} and the coordinate x corresponds to g_{ij}), one notes the absence of an energy E. Such energy is the time rate of change of action:

$$E = -\,\partial S/\partial t \tag{18}$$

and is related to *dispersion* and *propagation*:

$$E = H(p, x) \tag{19}$$

This energy of propagation is neglected in the dynamics of general relativity because all information about time which is relevant to that theory is evolution of space-time forms a trajectory. This space is called "superspace" and every point of superspace corresponds to a three-geometry $^{(3)}G$. Fisher (1970) has constructed a model superspace which is the quotient space Riem(M)/Diff(M). Riem(M) is the space whose points are nonsingular, Riemannian metrics on M, and Diff(M) is the group of diffeomorphisms of M acting on Riem(M) to transform each three-metric g_{ij} according to the usual transformation law under coordinate transformations. $S(M)$ is not itself a manifold and two superspaces, $S(M_1)$ and $S(M_2)$, over two manifolds of different topologies are separate spaces. The DeWitt propagation law:

$$g^{1/2}\left[\tfrac{1}{2}g_{ij}g^{kl} - g_{ik}g^{jl}\right](\partial S/\partial g_{ij})(\partial S/\partial g_{kl}) + g^{1/2}R \tag{20}$$

does not allow propagation from $S(M_1)$ to the differing $S(M_2)$. In order to construct a hybrid superspace which would allow propagation between differing topologies, DeWitt (1972) has obtained an "extended" superspace. The points of extended superspace carry *more* information than the three-geometries; information which is related to topological transition.

In the next section we exploit a result of Kiehn (1974) that dissipative systems may be associated with topology transition. If the dispersion energy neglected by geometrodynamics is linked to internal time, which is the basis of irreversible dissipative systems, one links internal time to topology transition via propagation in extended superspace. It is expected that the extended superspace would then consist of points which carry more information than the three-geometries. The additional information would be related to an aspect of time "irrelevant to general relativity." This internal time expresses the evolution of space-time geometry–the evolution of the universe–differently than dynamical clock time.

4. INTERNAL TIME AND IRREVERSIBLE DISSIPATIVE SYSTEMS

The theory of irreversibility of Prigogine et al. (Misra et al., 1979) allows particular entropy increasing evolutions–previously described by stochastic methods–to be connected to deterministic dynamics via a non-unitary similarity transformation Λ. Thus, the time evolution of a distribution function ρ, given by the Liouville equation (deterministically)

$$i\partial\rho/\partial t = L\rho \tag{21}$$

is acted upon by the transform $\rho \to \hat{\rho} = \Lambda\rho$ to give

$$i\partial\hat{\rho}/\partial t = \Phi\hat{\rho}, \qquad \Phi = \Lambda L \Lambda^{-1} \tag{22}$$

The nonunitary requirement on Λ makes the functional

$$\Omega = \int |\rho|^2 \, d\mu \tag{23}$$

a Lyapounov function. A dynamical system exhibiting a high degree of instability, thus admitting a Lyapounov variable, is the K flow. The geodesic flow for the Robertson–Walker universe is an example of K flow and is rapidly divergent.

For K flow there exists an operator time T which satisfies the commutation relation

$$i[L, T] = I \tag{24}$$

One can obtain a Lyapounov variable which is a monotonically decreasing function of T,

$$M = M(T) \tag{25}$$

and gives the nonunitary transformation Λ:

$$\Lambda = M^{1/2} \tag{26}$$

The operator M acting on the distribution functions ρ then gives a monotonically decreasing inner product

$$\langle \rho, M\rho \rangle = \int \rho^* M \rho \, d\mu \tag{27}$$

This implies the commutator

$$i[L, M] \leqslant 0 \tag{28}$$

The dynamical (Lyapounov) variable M expresses the intrinsic irreversibility of dynamical evolution for the case of K flow. This internal time operator M does not exist for all systems, but for the evolution of the universe given by the Robertson–Walker metric, M can be introduced. The dynamical evolution of that cosmological model is thus equivalent to a dissipative irreversible evolution under the similarity transformation Λ.

Kiehn (1974, 1975) has made an extension of Hamilton's principle to include *dissipative systems* by describing those systems with a closed integral of action which is a parameter-dependent conformal invariant of motion. Conservative systems are described by an independent absolute invariant of motion. He has shown that trajectory continuity is explicitly dependent upon a (conformal) dissipation function Γ and the conformal function is an invariant of orientation-preserving deformations. Thus dissipative systems may be associated with topological transition.

The vanishing of the Lie derivative of an object w,

$$\pounds_V w = 0 \tag{29}$$

means that w is invariant with respect to propagation down the trajectories of the vector field $V = (p_u, f_u, 1)$. If the object is the action S and S admits an integrating factor β such that βS is an absolute invariant with respect to a parameterized vector field γV in the above sense, i.e.,

$$\pounds_{\gamma V} \beta S = 0 \tag{30}$$

then the action is *conformally invariant*:

$$\pounds_{\gamma V} S = \Gamma S \tag{31}$$

Nonadiabatic systems are treated by the introduction of this conformal dissipation factor Γ. The conformal invariant may be expressed

$$-\Gamma/\gamma = \partial(\ln\beta)/\partial t + v^\mu \partial(\ln\beta)/\partial q^\mu + f_\mu \partial(\ln\beta)/\partial p_\mu \tag{32}$$

where, $\{q^\mu, p^\mu, \tau\}$ is the phase space.

The Lie derivative may be constructed in terms of the interior product (i) and exterior derivative (d) operators

$$\pounds_{\gamma V} S = i(\gamma V)\, dS + di(\gamma V) S \tag{33}$$

Kiehn has focused attention on the form of the first term to obtain constraints on the vector field V which leave the closed integral of action S conformally invariant for a given parameterization γ:

$$\mathscr{L}_{\gamma V} \oint S = \oint \mathscr{L}_{\gamma V} S = \oint \Gamma S \tag{34}$$

Thus

$$\oint \{\pounds_{\gamma V} S - \Gamma S\} = 0 \tag{35}$$

and by equation (33)

$$\oint \{ i(\gamma V) dS + di(\gamma V) S - \Gamma S \} = 0 \tag{36}$$

The middle term is an integral of a perfect differential and vanishes over a closed cycle z

$$\oint di(\gamma V) S = 0 \tag{37}$$

so we are left with

$$\oint \{ i(\gamma V) dS - \Gamma S \} = 0 \tag{38}$$

By DeRham's theorem the integrand here is a perfect differential $-dP$, vanishing over the closed cycle z so

$$i(\gamma V) dS - \Gamma S = -dP \quad \text{or}$$

$$i(\gamma V) dS = \Gamma S - dP \tag{39}$$

This constraint gives a generalized vector field of which the *extremal field*,

$$i(\gamma V) dS = 0 \tag{40}$$

is a special case. Furthermore, equation (39) defines an equivalence class of vector fields which are generators of a homotopy, where $i(\gamma V)$ is the homotopy operator. In this sense then a conformal invariant Γ is an invariant of an orientation-preserving deformation. Thus dissipation may be linked with topological change (Kiehn, 1974, 1975).

The introduction of internal time to the problem of cosmological evolution, for the Robertson–Walker universe, entails an evolving distribution of particles (the cosmological fluid) which is highly unstable. This rapid divergence can also be illuminated via Kiehn's extension of Hamilton's principle.

Consider the evaluation of equation (39) for the vector field $V = (v^\mu, f_\mu, 1)$. The Hamiltonian equations are

$$v^\mu = \partial \mathscr{H} / \partial p_\mu + (1/\gamma)(\partial P / \partial p_\mu) \tag{41}$$

$$f_\mu = -\partial \mathscr{H} / \partial q^\mu - (1/\gamma)(\partial P / \partial q^\mu) + (\Gamma/\gamma) p_\mu \tag{42}$$

and

$$\Gamma\{ p_\mu(\partial \mathscr{H}/\partial p_\mu) - \mathscr{H} \} - (\partial P/\partial \tau + \{ P, \mathscr{H} \}) = 0 \tag{43}$$

where $\{ P, \mathscr{H} \}$ are Poisson brackets and (p_μ, q^μ, τ) is the phase space.

The variation of the Hamiltonian $\mathscr{H}$ down the nonadiabatic flow γV is the Lie derivative

$$£_{\gamma V}\mathscr{H} = \{ \Gamma(\mathscr{H})/\gamma + (1/\gamma)(\partial P/\partial \tau) + \partial \mathscr{H}/\partial \tau \} \tag{44}$$

The density on phase space (p_μ, q^μ, τ) is

$$m = \mu\left(q^\mu, p_\mu, \tau\right) dq^\mu \, dp_\mu \tag{45}$$

and its variation $£_{\gamma V}(m)$ involves a continuity term β. This term is defined as

$$\beta = \left\{ \frac{\gamma}{\mu} \frac{d\mu}{d\tau} + \frac{\partial(\gamma v^\mu)}{\partial q^\mu} + \frac{\partial(\gamma f_\mu)}{\partial p_\mu} \right\} \tag{46}$$

and the condition of continuity, $\beta = 0$, may be combined with the Hamiltonian equations (41)–(43) to require

$$\frac{\gamma}{\mu} \frac{d\mu}{d\tau} + \{ \gamma, \mathscr{H} \} + \left(\Gamma + p_\mu \frac{\partial \Gamma}{\partial p_\mu} \right) = 0 \tag{47}$$

It should be noted that the perfect differential term P does not appear explicitly here. Further, for the case $\{ \gamma, \mathscr{H} \} = 0$ and $p_\mu \partial \Gamma / \partial p_\mu = 0$ we get

$$d(\ln \mu)/d\tau = -\Gamma/\gamma \tag{48}$$

Thus the conformal dissipative factor Γ is related to a rapidly diverging evolution of the distribution μ.

Extending this result to the cosmological problem is possible when we consider the phase space (p_μ, q^μ, τ) goes over to a "*super*" *phase space* $(\pi_{ij}, g^{ij},\lambda(\tau))$ where the "coordinates" are the three geometries ${}^{(3)}G$ and the "momenta" are the superspace momentum π_{ij}. The Hamiltonian $\mathscr{H}$ becomes the "super-Hamiltonian" H and the density refers to geodesic spacing for particles moving with the cosmological fluid. As in Section 2, we obtain a rapidly divergent evolution.

The link between topology and dissipative systems suggests a geometrical counterpart to internal time M of the universe. The dissipative energy of the evolving aging universe may be related to dispersion of the propagation in grand superspace of many topologies. These aspects of time will be examined next.

5. AGE, GEOMETRY, AND ENERGY

The evolution of the universe may be described with an emphasis on dynamical time—clock time—via general relativity and its modern formulation in geometrodynamics. In this conventional description, the dynamic entity is three-dimensional space with intrinsic and extrinsic curvature: the three-geometry ${}^{(3)}G$. The arena for the dynamics is superspace in which every point is a different configuration of ${}^{(3)}G$. Associated with this geometrodynamics is a super-space momentum π_{ij}. The "time" variable of geometrodynamics is contained in the extrinsic curvature K_{ij} of the ${}^{(3)}G$ (that is, the lapse and shift functions N and N^i) and is somewhat like classical clock time. Classical clock time is an independent parameter of motion. Similarly, the lapse and shift functions, N and N^i, are independent parameters in geometrodynamics. As we move along a trajectory in superspace, they allow us to freely choose to move forward or backward, faster or slower. Furthermore, there is no time component of superspace momentum π_{ij} $(i, j=1,2,3)$, just as there is no time component in the classical momentum p_i of a particle. Such a time component π_{44} would entail the use of "four-geometry" ${}^{(4)}G$, but this is not possible in geometrodynamics since, "all information about time which is relevant to general relativity is contained in the three-geometry ${}^{(3)}G$." Therefore this additional time component is neglected in relativity. The energy conjugate to the time component of momentum is also neglected.

It is tempting to expand upon geometrodynamics and make a generalization like the theories of special and general relativity. Let us suppose that irreversibility is a geometric feature of the space-time manifold in a sense similar to the incorporation of gravity into space-time by general relativity. The theory is: the irreversibility of the evolving universe is due to space-time topology Γ while gravity, according to relativity, is due to space-time curvature, $G_{\mu\nu}$. In other words, the internal time M which characterizes irreversible cosmological age has its geometric counterpart in space-time topology.

One of the features of a geometric counterpart to internal time M is the extension of Wheeler's notion of superspace to a grand superspace of many topologies. Of all possible evolutions of the universe, the allowable evolutions would be restricted to those trajectories in grand superspace which preserve the conformal invariant Γ. Each point of the extended superspace contains more information than the three-geometry alone; specifically information about topology transition. In turn, this additional information—which has its origin in cosmological age (space-time topology)—can be assigned to the time component of momentum π_{uv}. This assignment represents an aspect of time (duration) which is meaningful and manifest in irreversibility yet different from dynamic (clock) time. The associated energy conjugate to the time component π_{44} of the momentum may be identified with the dispersion. These details will be discussed in another publication.

The treatment of irreversible evolution (at least for the case of the aging cosmological fluid of the Robertson–Walker universe) in terms of space-time geometry does not resort to the statistical notion of entropy. That quantity is a macro quantity which emerges only for many particles. It does not enter conventional dynamics for a few particles, and it is interesting to ask whether the proposed geometrical basis of irreversibility has any "entropy-like" effect for the dynamics of a few particles. According to the theory presented here, this is equivalent to asking about the relation of local features of space-time geometry to global (topological) features.

The Lorentz signature on the metric $(+\ -\ -\ -)$ is a global feature of space-time related to the orientation of hypersurfaces (Hawking and Ellis, 1973). If g_{ij} is a metric in the three-geometry ${}^{(3)}G$, induced by a metric g_{uv} of the four-geometry in which the ${}^{(3)}G$ is imbedded, then g_{ik} is Lorentz when $g^{uv}\eta_u\eta_v > 0$. This gives a three-geometry ${}^{(3)}G$ which corresponds to a timelike hypersurface orientation. (Other orientations are possible.) Recall the key idea which allowed the link of irreversibility and geometry: the identification of the dissipative conformal invariant Γ as an invariant of an orientation-preserving deformation. We conclude that there is a connection between the metric signature and irreversibility. The Lorentz signature is also a "local" feature of space-time associated with the causal ordering of events, so we recognize the local manifestation of irreversibility in the causal quality of interactions of arbitrarily few particles.

This connection of irreversibility and causality through topology may be seen in the following way. A point 0, and its arbitrary small neighborhood, U_0, is located from another point P, and its small neighborhood, U_p, by the interval s. Let a space be defined as the union of the neighborhoods

$$X|x \in U_0 \cup U_p \tag{49}$$

Let causal connection by a light signal of U_0 and U_p determine the topology, $S(I)$, on the space. An isotope (X, S) is formed. A certain time interval, t_s, is required for a light signal to connect points of the neighborhoods

$$t_s = s/c \tag{50}$$

(where c is the speed of light). At a time $t < t_s$ (here is the dynamical view of time as moments), the points of one neighborhood cannot be connected by light signal with the points of the other neighborhood, so a topology, $S1$, is established on X:

$$\forall t \ni t < t_s \exists U_0 \cap U_p = \varnothing \text{ then } (X, S1) \tag{51}$$

At a later time $t > t_s$, it is possible to connect points of the two neighborhoods with a light signal, so a new topology, $S2$, on X is formed

$$\forall t \ni t > t_s \exists U_0 \cap U_p \neq \varnothing \text{ then } (X, S2) \tag{52}$$

Thus the time interval $\Delta t = t_s$ is a transition of topology:

$$\Delta t \equiv (X, S2) \rightarrow (X, S2) \tag{53}$$

6. CONCLUSION

We have examined the internal time operator M for the Robertson–Walker universe and proposed its geometric counterpart. This gives a theory of cosmological irreversibility due to the geometric (topological) structure of space-time. By this account, the dynamics of general relativity appears as an approximate case assuming the physical irrelevance of irreversibility and topology. The possibility of this geometrical significance for internal time is inviting. Internal time differs profoundly from clock time, but if it could be incorporated in a geometric "world view," along with gravitation, we would arrive at a deeper and broader understanding of the unity of time and the world.

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